REPORT JUNIOR TRIMESTER PROGRAM TOPOLOGY

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The authors of this report were the members of one of the groups invited to the Junior Trimester Program on Topology at the Hausdorff Institute for Mathematics from September to December 2016. In this report, intended to be readable by non-specialists, we explain the work we accomplished during this program.

Our project is inspired by Grothendieck's "Esquisse d'un programme" [Gro97]. In this highly influential text, Grothendieck suggests that one should be able to study the absolute Galois group of $\mathbb{Q}$ via its action on objects coming from low-dimensional topology.

The absolute Galois group of $\mathbb{Q}$. Recall that an algebraic number is a complex number that is a solution of a polynomial equation with rational coefficients. These numbers form a subring of the complex numbers. The **absolute Galois group of** $\mathbb{Q}$ is by definition the group of automorphisms of that ring. Very little is known about this group and its understanding is one of the major problems of number theory. It is an uncountable group but paradoxically it is very hard to construct elements of that group. In fact the only non-trivial element with a known explicit description is complex conjugation.

Étale fundamental group and Galois representations. Recall that the **fundamental group** of a based connected topological space X is by definition the group of homotopy classes of loops on that space starting and ending at the base point. The main theorem in covering spaces theory is that the category of covering spaces of X is equivalent to the category of sets equipped with an action of the fundamental group. Grothendieck and his school noticed that although the notion of loops in an algebraic variety makes little sense, there is a purely algebraic description of the covering spaces of an algebraic variety. From this observation one can define the so-called **étale fundamental group** of a complex algebraic variety which is isomorphic to the profinite completion of the topological fundamental group (the profinite completion comes from the fact that only the finite covering spaces can be defined algebraically). In fact the theory uses nothing about the complex numbers and works over any field. From this fact, one obtains the striking result that the absolute Galois group of $\mathbb{Q}$ acts naturally on the étale fundamental group of any complex algebraic variety that is defined over $\mathbb{Q}$. This action can be highly non-trivial. In particular, it was proven by Belyi that this action is faithful on the étale fundamental group of a variety as simple as the projective line with the points 0 1 and ∞ removed.

Moduli spaces of Riemann surfaces of genus zero. The group Γ_0^n is defined as the mapping class group of a Riemann surface of genus zero with n fixed boundary components. Its classifying space $B\Gamma_0^n$ can be interpreted as the classifying space for bundles of such surfaces. It turns out that this space can be given a purely algebraic description : there exists an algebraic variety defined over $\mathbb{Q}$ whose topological space of complex points has the homotopy type of $B\Gamma_0^n$. From this observation and the previous paragraph, one gets an action of the absolute Galois group of $\mathbb{Q}$ on the profinite completion of the group Γ_0^n.

Operads and cyclic operads. The collection of spaces $\{B\Gamma_0^n\}_{n\in\mathbb{N}}$ has a rich algebraic structure. For any pair $(i,j)\in\{1,\dots,n\}\times\{1,\dots,m\}$, there is a gluing map

$$g_{i,j} : B\Gamma_0^n \times B\Gamma_0^m \to B\Gamma_0^{n+m-2}$$

that can be geometrically interpreted as the gluing of a genus zero Riemann surface with n-boundary components to one with m boundary components along the boundary components labelled i and j. The result of this operation is a Riemann surface of genus zero with $n+m-2$ boundary components. These operations satisfy a form of associativity as well as a compatibility with the obvious symmetric group actions on the spaces $B\Gamma_0^n$. Such an algebraic structure is called a **cyclic operad**. If one restricts to the gluing maps $g_{i,j}$ with $j=1$ and $i\neq 1$, one gets the structure of an **operad**. This operad is homotopy equivalent to a famous operad called **the framed little disks operad**.

Our work. As we said, the profinite completion of the mapping class groups Γ_0^n supports an action of the absolute Galois group of $\mathbf{Q}$. In the paper [BHR17] written for the most part during our stay at the HIM, we prove that this action is compatible with the operadic gluing maps $g_{i,j}$ with $j=1$ and $i\neq 1$ defined in the previous paragraph and defines a faithful action of the absolute Galois group of $\mathbb{Q}$ on the profinite completion of the framed little disks operad. Even though the spaces $B\Gamma_0^n$ have the homotopy type of algebraic varieties, it is not at all clear that the gluing maps are homotopic to algebraic maps which makes our result not obvious. In fact our method for proving this result it to completely compute the group of homotopy automorphisms of the operad of profinitely completed moduli spaces $\{B\Gamma_0^n\}_{n\in\mathbb{N}}$. We show that the resulting group is the so-called **profinite Grothendieck-Teichmüller group**. This group was introduced by Ihara in [Iha94] as an approximation of the absolute Galois group of $\mathbb{Q}$ coming from Belyi's theorem and geometric considerations. In particular Ihara proves that the absolute Galois group of $\mathbb{Q}$ injects into the Grothendieck-Teichmüller group.

Furture work. As explained in the previous paragraph, we computed the automorphisms of the operad of moduli spaces as opposed to the richer structure of cyclic operad. We intend to compute the automorphisms of the cyclic operad in future work. We conjecture that the resulting group will again be the Grothendieck-Teichmüller group. A much more ambitious project would be to compute the automorphisms of the **modular operad** of moduli spaces. This is a very complicated object that encodes all the possible gluing of Riemann surfaces along boundary components without any restriction on the genus.

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FINAL REPORT - THE FARRELL–JONES CONJECTURE

DANIEL KASPROWSKI, MARK ULLMANN, CHRISTOPH WINGES, AND JIANCHAO WU

We participated in the Junior Hausdorff Trimester Program *Topology* from September to December 2016. Our project title was "The Farrell–Jones conjecture". Daniel Kasprowski, Mark Ullmann and Christoph Winges stayed for the whole program while Jianchao Wu stayed from October 16 until September 20.

The Farrell–Jones conjecture. The Farrell-Jones conjecture is an important conjecture, connecting algebra, algebraic topology and geometric topology. It implies many other important conjectures, including the Borel conjecture about the topological rigidity of closed aspherical manifolds [BL12a], the Bass conjecture, the Novikov conjecture and the Kaplansky conjecture, see [BLR08c].

The Farrell–Jones conjecture was first formulated in [FJ93]. It has been proved for a large class of groups, for example hyperbolic and CAT(0)–groups, see Bartels and Lück [BL12a, BL12b], Bartels, Lück and Reich [BLR08a, BLR08b] and Wegner [Weg12], virtually solvable groups, see Wegner [Weg15], and lattices in virtually connected Lie groups, see Bartels, Farrell and Lück [BFL14] and Kammeyer, Lück and Rüping [KLR16]. Recently, Bartels [Bar17] showed that the Farrell–Jones conjecture holds for groups which are relatively hyperbolic to groups satisfying the conjecture.

Seminar. We organized a weekly learning seminar on the Farrell–Jones conjecture. Some of the talks were given by ourselves while others were given by topologists from Bonn. We also participated in the Trimester Seminar organized together by all groups of the Junior Trimester Program.

Conference. From October 24 to October 28 we organized a Workshop on the Farrell–Jones conjecture with 11 invited lectures and 45 registered participants.

The A–theoretic Farrell–Jones conjecture. While the Farrell–Jones conjecture is often only formulated for algebraic K–theory and L–theory, it can also be formulated for Waldhausen's A–theory. Before the workshop, in [UW], Ullmann and Winges developed a setting analogous to the linear case to prove the following:

Theorem ([UW, Theorem 1.1])**.** *Let G be a virtually poly-$\mathbb{Z}$–group. Then G satisfies the A–theoretic Farrell–Jones conjecture.*

During the program Ullmann and Winges worked with Nils-Edvin Enkelmann, Malte Pieper and Wolfgang Lück from Bonn to extend this theorem to hyperbolic and CAT(0)–groups [ELP$^+$].

Kasprowski, Ullmann and Winges also collaborated with Christian Wegner to prove the A–theoretic Farrell–Jones conjecture for solvable groups [KUWW].

Injectivity of the assembly map. The Farrell–Jones conjecture implies that the assembly maps for the family of finite subgroups are split injective. In L–theory, the rational injectivity of the assembly map implies the Novikov conjecture. The integral split injectivity of the assembly map is called the *generalized integral Novikov conjecture*. It has strong ties with the Baum-Connes conjecture for operator K–theory.

In [Kas16, Kas15], the generalized integral Novikov conjecture for all subgroups G of virtually connected Lie groups and linear groups over commutative rings was proved under the assumption that there exists a finite dimensional model for $E_{\mathcal{F}in}G$. As for the Farrell–Jones conjecture it makes sense to ask whether these results hold in the A–theoretic setting.

Answering this question is part of an on-going collaboration of Kasprowski and Winges with Ulrich Bunke and Alexander Engel. The aim of this project is to extend the setting of motivic coarse homology by Bunke and Engel [BE] to an equivariant version and give suitable conditions for a coarse homology theory that imply split injectivity of the assembly map in this coarse homology theory. Many important discussion on this project happened while Engel and Bunke visited us at the Hausdorff Research Institut during the Junior Trimester Program. More specifically, we expect the following preprints to become available on the arXiv in the near future:

(1) [BEKWb] provides the necessary generalization of [BE] to the equivariant setting and introduces coarse algebraic K–homology as the main example to which the general theory applies.
(2) Building on the basic theory developed in [BEKWb], we set up an axiomatic framework to prove injectivity in [BEKWc]. The main result formulates a split-injectivity result for the forget-control map associated to an equivariant coarse homology theory under appropriate assumptions on the theory and the group under consideration. We also show that algebraic K–homology fits into this framework.
(3) The third preprint [BEKWa] shows that the main result of [BEKWc] can be applied to groups satisfying a suitable version of finite decomposition complexity.

Other induction theorems. A crucial ingredient for the proof of the main theorem in [UW] is an A–theoretic analog of Swan's induction theorem. This analog can be derived from a difficult theorem of Oliver about the existence of fixed-point free actions on disks. Akhil Mathew from the group "Interactions between Goodwillie calculus, chromatic methods, and unstable homotopy theory" and Winges have been able to obtain an alternative proof as well as variations of the induction theorem. Instead of Smith theory, the proof relies on an induction result for the K–theory of Green functors in conjunction with general facts from equivariant stable homotopy theory. The preprint [MW] will hopefully be available soon.

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Final Report: Real Trace Methods

This is the final report for the group *Real trace methods* at the Junior Trimester Program (JTP) at the Hausdorff Institute for Mathematics (HIM) in the fall of 2016. The members of or group were Emanuele Dotto (Sep. 5 - Dec. 22), Amalie Høgenhaven (Nov. 7 - Nov. 28), Kristian J. Moi (Sep. 5 - Dec. 15), Irakli Patchkoria (Sep. 21 - Dec. 22) and Sune P. Reeh (Sep. 5 - Dec. 22).

Composition of the group and research goals

Our research group resulted from a combination of two separate applications to the HIM, one by E. Dotto, A. Høgenhaven, K. Moi and I. Patchkoria to work on real algebraic K-theory and trace methods, and one by S. Reeh to work on fusion systems and transfers maps in stable homotopy theory. This joining of forces was very successful, broadening the perspective of the group and creating an atmosphere full of ideas and discussions.

Broadly speaking, the research goal for the group was to study questions arising in number theory, group theory and differential topology with methods of stable homotopy theory. One powerful method of this kind is algebraic K-theory and trace maps. Recent work of Hesselholt and Madsen [HM15] has made the formidable tools of equivariant stable homotopy theory available to study the involution on algebraic K-theory. A goal for our group was to extend these ideas far enough to start making explicit calculations. Another goal was to understand p-completion and loop transfer maps for classifying spaces of finite groups.

Activities

Our group organized a number of activities over the course of the trimester. We also benefited greatly from working with the other groups, especially the group *New perspectives in A-theory* whose interests overlapped significantly with ours.

Seminars and visitors

Together with the group *New perspectives in A-theory* our group organized a semi-weekly *K-theory seminar*. We invited Heng Xie from the University of Warwick to come on October 11-13 and had many visitors in connection with our two workshops. Our group also regularly attended the *A-theory group seminar* and worked with visitors to the A-theory group such as Nat Stapleton, Michael Weiss and Wolfgang Steimle.

Workshop: Hermitian K-theory and trace methods (November 7-11)

The group organized a workshop on interactions between algebraic and Hermitian K-theory with topological Hochschild homology. It consisted of individual talks as well as four mini-courses, given by Bjørn Dundas, Lars Hesselholt, Oliver Röndigs and Marco Schlichting. The mini-course of Hesselholt was given under *Felix Klein lectures* umbrella with title *Around topological Hochschild homology*, and continued into the following week. To make the event more accessible for junior mathematicians in need of travel funding the group received financial support from the SPP 1786 grant and in addition some participants were funded by the Felix Klein lecture grant. The workshop had 77 registered participants, this number includes the guests at the Junior Trimester Program.

Workshop: Fusion systems and equivariant algebraic topology (November 21-24)

S. Reeh organized this workshop on the role of fusion systems in algebraic topology. It consisted of individual lectures as well as two lecture series by Radu Stancu and Jesper Grodal. The individual talks, most of them given by young researchers, covered a range of topics involving fusion systems in topology, including homology calculations, group actions on spheres and chromatic homotopy theory. The workshop had 44 registered participants, this number includes the guests at the Junior Trimester Program.

Projects and papers

Below are some projects and papers that members of the group worked on during the JTP.

Project: On real THH This on-going joint project of E. Dotto, K. Moi, I. Patchkoria and S. Reeh, was one of the focal points of our group at the trimester program. We prove the equivalence of two models for real topological Hochschild homology (THR) and derive a formula for its $\mathbb{Z}/2$-geometric fixed points. Using the formula we compute the homotopy types of $\mathrm{THR}(\mathbb{F}_p)$ and of the geometric fixed points of $\mathrm{THR}(\mathbb{Z})$.

Paper: Comparing cyclotomic structures on different models for topological Hochschild homology [DMP+17] This joint paper of E. Dotto, C. Malkiewich, I. Patchkoria, S Sagave and C. Woo, proves the equivalence of the cyclotomic spectra arising from the Bökstedt construction of THH and the cyclic bar construction in orthogonal spectra.

Paper: Hermitian K-theory of Mackey functors and a reformulation of the Novikov conjecture [DO17] This paper of E. Dotto and C. Ogle introduces *Hermitian Mackey functors* and their real algebraic K-theory. Building on the results of the *On real THH*-project it gives a reformulation of the Novikov conjecture in terms of the trace map to THR.

Paper: A formula for p-completion by way of the Segal conjecture [RSS17] This joint paper of S. Reeh, T. Schlank and N. Stapleton describes the p-completion functor on stable maps between classifying space purely in terms of fusion data and Burnside modules.

Project (started at HIM): Real algebraic K-theory and L-theory This project of E. Dotto, K. Moi and T. Nikolaus defines real algebraic K-theory of stable ∞-categories with quadratic functor and studies its connection to the L-theory of such categories as recently defined by Jacob Lurie.

Project (started at HIM): A homotopy coherent approach to fusion systems This project of C. Barwick, S. Reeh and R. Molinier attempts to construct a good ∞-categorical model for the orbit category of an abstract saturated fusion system.

The group would like to thank the HIM and its staff for their hospitality and the holders of the SPP 1786 grant and the Felix Klein lecture grant for the financial support provided to our workshop.

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New Perspectives in A-theory

Final report

The *New perspectives on A-theory* group participated in the first half of the Junior Trimester Program at the Hausdorff Institute for Mathematics (HIM) in Bonn, during the fall of 2016. The members of our group were Cary Malkiewich (SUNY Binghamton, UIUC at the time), Mona Merling (Johns Hopkins) and George Raptis (Regensburg). One of our intended group members, John Lind (Reed College) was not able to attend, but he very much influenced our research focuses as we worked together on the proposal for the program. We are very thankful to the HIM for giving us the opportunity to work in such a stimulating place and offering us outstanding working conditions. Being able to interact with other groups (in particular the *Real Trace Methods* group), members of the Bonn mathematical community and visitors to the HIM during the course of our stay has led to progress in our projects and to new collaborations.

Overview of our group's research focus

Waldhausen's algebraic K-theory of spaces $A(X)$ is an algebraic invariant of topological spaces with deep geometric content. When X is a smooth manifold, $A(X)$ contains not only the stable homotopy type of X but also the stable pseudo-isotopy theory of X. At the same time, since $A(X) = K(S[\Omega X])$ is a special case of the algebraic K-theory of ring spectra, Waldhausen's K-theory also contains arithmetic information and is useful in the study of the K-theory of ordinary rings.

The two main focuses of our group were (1) to understand the relationship between the A-theory transfer, the transfer on topological Hochschild homology and the Becker-Gottlieb transfer, and (2) to generalize A-theory to the case when the input space has a group action on it.

Activities

We organized a seminar with invited speakers and an internal joint seminar together with the the *Real Trace Methods* group in order to facilitate interaction between the two groups. Both seminars have been very well attended and gave us and the other participants the opportunity to learn about new developments related to our topics of interest and also to interact with other experts. Most of the invited speakers spent a whole week at the HIM, which gave us the chance to have in depth discussions and start new collaborations.

Resulting projects, papers, and collaborations

Some collaborative projects that we worked on at the HIM, and some discussions that we hope will continue and lead to full-fledged projects:

- A joint project of C. Malkiewich, M. Merling and G. Raptis that was started at the HIM and was our main focus as a group is aimed at understanding the A-theory transfer maps for perfect fibrations $E \to B$, in terms of the pieces of the splitting $A(X) \simeq \Sigma^\infty_+ X \times Wh(X)$. We discussed this project with members of the JPT program at the HIM, in particular with M. Ullman, and with M. Weiss and O. Sommer from U. Münster, who were invited speakers in our seminar. One of our group members, C. Malkiewich, has visited the HIM again in the summer of 2017 and has had the opportunity to discuss this project with W. Lück, which reinvigorated our efforts.

- Part of an ongoing joint project of M. Merling and C. Malkiewich on equivariant A-theory, was completed at the HIM. The resulting paper

  C. Malkiewich and M. Merling, *Equivariant A-theory*, `arXiv:1609.03429`

  gives two definitions of equivariant A-theory, one related to bivariant A-theory and one related to equivariant h-cobordisms. A follow-up paper will prove an equivariant version of the stable parametrized h-cobordism theorem.

- G. Raptis and W. Steimle have worked during Steimle's visit to the HIM as an invited speaker in our seminar on a project on giving a "cobordism model" for Waldhausen's $S_\bullet$-construction. A paper on this is forthcoming.

- C. Malkiewich started a collaboration at the HIM on comparing the cyclotomic spectra arising from the Bökstedt construction of THH and the cyclic bar construction in orthogonal spectra, which led to the paper

  E. Dotto, C. Malkiewich, I. Patchkoria, S Sagave and C. Woo, *Comparing cyclotomic structures on different models for topological Hochschild homology*, `arXiv:1707.07862.`

- While N. Stapleton and J. Noel were visiting the HIM as invited speaker in our seminar, they discussed with C. Malkiewich and M. Merling a project on connecting of a spectral version of the character map from representation theory that Malkiewich and Merling were thinking about with the Hopkins-Kuhn-Ravenel character and Stapleton's transchromatic character map.

- E. Dotto, M. Merling and K. Moi have discussed a project started by Dotto and Merling on proving an equivariant version of Waldhausen's additivity theorem. Discussions with I. Patchkoria on this subject have been very illuminating.

Taking advantage of the wonderful working conditions at the HIM and the opportunity to discuss ideas with the other members of the program, our group members have also individually worked on the following projects and papers (some with collaborators who were not at the HIM at the time):

- C. Malkiewich was working remotely with K. Ponto on a project on building a connection between homotopy theory and dynamics. A paper

  C. Malkiewich and K. Ponto, *Periodic orbits and topological restriction homology*

  is in progress, which will set the stage for this new direction in algebraic topology.

- G. Raptis was working on the final stages of the manuscript

  G. Biedermann, G. Raptis and M. Stelzer, *The realization space of an unstable coalgebra*, to appear in *Astérisque.*

- G. Raptis worked on a devissage theorem for Waldhausen K-theory and discussed some aspects of his work with members of the program. A paper by Raptis is forthcoming.

- M. Merling was working on finishing the paper

  J.P. May, M. Merling and A. Osorno, *Equivariant infinite loop space theory, I.The space level story*, `arXiv:1704.03413`

  on equivariant infinite loop space theory with collaborators who were not at the HIM. She had the opportunity to discuss this project with S. Schwede and learn his invaluable insights, which led to another paper about to be posted

  B. Guillou, J.P. May, M. Merling and A. Osorno, *Equivariant infinite loop space theory II. The multiplicative Segal machine*

- M. Merling also worked on finishing the paper

  A. Beaudry, K. Hess, M. Kedziorek, M. Merling and V. Stojanoska, *Motivic homotopical Galois extensions*, to appear in *Topology and its Applications*, `arXiv:1611.00382`.

  on motivic Galois extensions with collaborators who were not at the HIM was in its final stage while I was there. Being able to discuss certain aspects of it with participants at the HIM was very helpful.

Junior Trimester Program "Topology"

09/2016 - 12/2016

Working group "ℓ^2-invariants"

Final report

G. Herrmann, H. Kammeyer, A. Kar, S. Kionke, J. Raimbault

Our group was concerned with topological and group theoretic areas where ℓ^2-invariants have proven to be useful tools. In the course of the trimester, we were reading about twisted ℓ^2-torsion invariants for 3-manifolds, cost of measurable equivalence relations and approximation of ℓ^2-Betti numbers and rank gradient under the Farber condition. As research projects, we mainly discussed the two following problems:

- The approximation of Novikov–Shubin invariants by finite dimensional counterparts;
- The profinite invariance of ℓ^2-invariants, in particular ℓ^2-torsion.

For the first of these we wanted to study whether, in a sequence of finite groups "approximating" an infinite discrete group G, the rate of decay of spectral gaps (in terms of the order of the groups) are related to the Novikov–Shubin invariants. This is related to determinant approximation in the sense of Lück and in the only case where the latter is known to hold (for $G = \mathbb{Z}$) one of us [1] proved such a result, which was one of our motivations. We managed to get a rudimentary result in another approximation setting used by Li–Thom (we did find a much more elementary proof of one of their results). We think there might be more interesting work to be done on this, and that there might be possible (though not straightforward) relations to Lück's original problem.

The motivation for the second question is that for 3-manifolds, ℓ^2-torsion is proportional to volume and the question has gained interest recently whether two 3-manifolds have the same volume if their fundamental groups have isomorphic profinite completions. One idea we had in this context is to ask if volume is a profinite invariant for S-arithmetic subgroups in (possibly differing) semisimple algebraic groups of higher rank. The advantage in this setting is that for most higher rank S-arithmetic groups the congruence subgroup property is known to hold true. As a consequence, the profinite completion of an S-arithmetic group gives information on the k_v-points of the surrounding algebraic group for finite places v. This could potentially allow the conclusion of equal volume employing the Prasad volume formula. We are still working on this question and plan to publish some partial results in the near future [2]. A complete answer is still not in reach due to many technical difficulties, in particular so in the arising Bruhat-Tits theory. Some more articles have benefited from the program [3, 4].

In October we had our workshop entitled "New directions in L^2-invariants" with international guests and interesting talks on recent developments. It was a successful event to which we have obtained positive and grateful feedback from the participants. Another regular activity was our "L^2-seminar" with both local and non-local speakers which in addition to the HIM participants has also attracted researchers from both the MPI and the Mathematical Institute.

We also benefited from the activities of the other groups. For example, some of us attended various sessions of the Farrell–Jones seminar and of the 4-manifold group. The daily afternoon meetings were a welcome opportunity to keep in touch with the other groups.

Administrative staff was extremely well-organized, efficient and helpful before, during and after the program. The Hausdorff Institute has provided all necessary infrastructure and a stimulating atmosphere for the program. The junior version of the trimester programs is particularly enjoyable as for most of us it was the first opportunity to carry out an independent research program on our own responsibility. We all agree that we enjoyed the event as a whole and hope to be back at HIM at later stages of our career.

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[3] S. Kionke, *Characters, L2-Betti numbers and an equivariant approximation theorem* (2017), eprint. arXiv: 1702.02599.

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From September to December 2016, as part of the Junior Topology Trimester at the Hausdorff Institute for Mathematics, we participated in a research subgroup of the larger activity focused on low-dimensional topology. The participants of our group were: Christopher Davis, Peter Feller, Min Hoon Kim, Duncan McCoy, Jeffrey Meier, Allison Miller, Jung Hwan Park, Mark Powell, Patrick Orson, Matthias Nagel, and Arunima Ray.

Amongst all manifolds, those of dimension 4 remain among the least understood. Our research group studied 4-manifolds and related topics, especially the embeddings of spheres and discs inside 4-manifolds: the closely related theories of knot and link concordance, and 2-knots. We also took the opportunity to learn more about key tools in the study of 4-manifolds.

Our group was based in four offices in the HIM annex, which allowed frequent interactions and a vibrant and exciting atmosphere. We also met formally every Monday afternoon between 3-4pm to present new problems, progress on current problems, or to let the group know what research was planned for the week. The discussions sparked by these sessions led to several of the new projects outlined below.

The high quality coffee and cake, and the opportunity this provided to meet and interact with other young mathematicians from many of the different aspects of topology, was much appreciated. Likewise, the weekly joint seminar, in which a group leader explained the background to the research of his or her group, was extremely instructive.

Learning seminars. We ran four learning seminars during the semester, two during September and October and two in November and December. The goal of these was to increase the knowledge of the group in subject areas in which certain members have a particular speciality. A lot of recent work in the field uses Heegaard Floer homology and other gauge theory techniques, and all the members of our group make use of these tools. However some topological techniques were less well known to recent graduate students in the field, and this was rectified by some of the learning seminars. Moreover we learnt about trisections of 4-manifolds, which is a fruitful new method for describing and manipulating smooth 4-manifolds. Here are our learning seminars and their organisers:

(1) Filtrations of knot concordance and signature obstructions (CD, AM, MHK, AR).
(2) Trisections of 4-manifolds (JM).
(3) Surgery theory and applications to 4-manifolds and knot theory (PO and MP).
(4) Freedman's disc embedding theorem (MP and PF).

The Freedman learning seminar gave us the occasion to revisit lecture notes from the 2013 MPIM 4-manifolds semester, and rewrite large portions of it, producing a new version of the proof of Freedman's theorem. This should be a very useful service to the community. All of our group participated in this rewriting process, each assigned a chapter to improve. We also re-watched and discussed the videos from the 2013 lecture series as part of the learning seminar.

In addition, we attended learning seminars of the Farrell-Jones conjectures group, and the conference of the $L^{(2)}$-invariants group, which were the two closest research groups to ours.

Conference. We organised, with the help of the MPIM, a hugely successful conference on 4-manifolds and knot concordance, with many of the leading figures in the field (ourselves excluded) giving talks. Much new interaction was engendered.

Visitors. The following people visited us for between a week and a month and contributed to the programme through collaborations, discussions, giving seminars and guest lecture series: Paolo Aceto, Jae Choon Cha, Matt Hedden, Shelly Harvey, Ana Lecuona, Jonathon Hillman, Lukas Lewark, Andrew Lobb, Stefan Friedl, Alex Zupan, Juanita Pinzon Caicedo, Kyungbae Park, Stefan Behrens, Boldizsar Kalmar, Marco Golla.

New mathematics. Here are some of the scientific outcomes of this semester.

(1) CD–MN–PO–MP: used a refinement of Milnor's triple linking numbers to determine precisely when two links admit homeomorphic surface systems. The article is very near completion.
(2) CD: studied the solvable filtration and links in homology spheres. There is a natural extension of the solvable filtration of Cochran-Orr-Teichner to the setting of knots and links in homology spheres. The main result is that for every $n \in \mathbb{N}$, every link in a homology sphere is n-solve-equivalent to a knot in S^3. The article is very near completion.
(3) CD–MN–JHP–AR: studied smooth and topological concordance of knots in $S^1 \times S^2$. They showed that there is a unique smooth concordance class of knots with winding number one. Additionally, they demonstrated the distinction between the smooth and topological concordance of knots in $S^1 \times S^2$. A paper written, listed below.
(4) CD–JHP–AR: studied the effect of cabling operators on the n–solvable filtration of the knot concordance group and related this to the failure of Kauffman's conjecture on slice knots. They also studied a new filtration, the handlebody filtration, which arises naturally while studying the failure of Kauffman's conjecture. This work is in progress.
(5) DM: proved that a torus knot has only finitely many non-integer non-characterizing slopes. A paper written, listed below.
(6) DM and Lewark: calculated many new values of the smooth and topological slice genera for knots with 11 and 12 crossings. This subsumed previous work by DM. A paper written, listed below.
(7) PF and Lewark: studied slice genera of knots using the Seifert form. A paper written, listed below.
(8) MHK and Cha: showed that the graded quotient of the bipolar filtration $\{\mathcal{T}_n\}_{n=0}^{\infty}$ has infinite rank at every stage greater than one. The article is very near completion.
(9) MHK–JHP and Krcatovich: constructed infinitely many 2-component links with unknotted components which are topologically concordant to the Hopf link, but not smoothly concordant to any 2-component link with trivial Alexander polynomial. A paper written, listed below.
(10) MHK and Hedden and K. Park: found irreducible 3-manifolds whose fundamental groups have weight one, but they cannot be obtained by doing 0-surgery on a knot in S^3. These manifolds are not homology cobordant to any Seifert-fibred manifold. The article is very near completion.
(11) AM and Picirillo: proved that there exist non smoothly concordant knots with diffeomorphic zero traces. Proved that there exist invertible satellite maps on the smooth concordance group which do not act by connected sum with any fixed knot. A paper written, listed below.
(12) PF–MP: proved a criterion that implies boundary links are weakly topologically slice, making use of the topological surgery learnt in the seminars. Weakly slice links are the boundary of some locally flat planar surface in the 4-ball. They are working on extending it to other classes of links.
(13) AM–MP: computed metabelian Blanchfield forms and used this to provide a new computable slice obstruction for satellite knots. It also simplifies old proofs that certain knots are not slice. The article is very near completion.
(14) JM–MP: described a diagrammatic calculus of banded links in 4–manifolds, which represents embedded surfaces. It is based on a diagrammatic calculus due to Akbulut. In order to prove the fundamental results, we need embedded Cerf theory, which is the subject of another ongoing project of Powell with Borodzik and Teichner.
(15) JM and Zupan: studied trisections of 4-manifolds, and the related theory of bridge trisections of surfaces, which provides presentations of knotted surfaces in 4-manifolds. They related trisections to the generalized property R conjecture. Two papers, listed below.
(16) MN–PO–MP, with Friedl and with JHP: studied the problem of almost concordance of knots in 3–manifolds, which is concordance modulo local knots. They discovered new

obstructions and produced knots to realise those obstructions. They also exhibited the difference in the smooth and topological categories in this setting. Two papers written, listed below.

(17) MP, with Kasprowski–Teichner: continued work on a project to classify 4-manifolds up to stable diffeomorphism, which means allowing connected sum with $S^2 \times S^2$ or with $\mathbb{CP}^2$. A great deal of progress was made during the trimester, in particular on expanding the class of fundamental groups. The article is near completion.

Completed papers that arose from the trimester.

(1) Concordance of knots in $S^1 \times S^2$, by Christopher W. Davis, Matthias Nagel, JungHwan Park and Arunima Ray. arXiv:1707.04542.
(2) On classical upper bounds for slice genera, by Peter Feller and Lukas Lewark. arXiv:1611.02679.
(3) Satellites and concordance of knots in 3-manifolds, by Stefan Friedl, Matthias Nagel, Patrick Orson and Mark Powell. To appear in Transactions of the AMS. arXiv:1611.09114.
(4) Links with nontrivial Alexander polynomial which are topologically concordant to the Hopf link, by Min Hoon Kim, David Krcatovich and JungHwan Park. arXiv:1703.10325.
(5) Non-integer characterizing slopes for torus knots, by Duncan McCoy. arXiv:1610.03283.
(6) Characterizing Dehn surgeries on links via trisections, by Jeffrey Meier and Alexander Zupan. arXiv:1707.08955
(7) Trisections and spun 4–manifolds, by Jeffrey Meier. arXiv:1708.01214
(8) Knot traces and concordance, by Allison N. Miller and Lisa Piccirillo. arXiv:1702.03974.
(9) Smooth and topological almost concordance, by Matthias Nagel, Patrick Orson, Jung Hwan Park and Mark Powell. arXiv:1707.01147.
(10) The round handle problem, by Min Hoon Kim, Mark Powell and Peter Teichner. arXiv:1706.09571.
(11) On calculating the slice genera of 11- and 12-crossing knots, by Lukas Lewark and Duncan McCoy. To appear in Experimental Mathematics. arXiv:1508.01098. (This paper was uploaded to the arXiv as a replacement of an unpublished note by the second author. The version from the trimester appeared in November 2016.)

HIM JUNIOR TRIMESTER TOPOLOGY 2016 FINAL REPORT

Interactions between Goodwillie calculus, chromatic methods, and unstable homotopy theory

Our research group, consisting of Rosona Eldred, Gijs Heuts, Akhil Mathew, and Lennart Meier, worked at the Hausdorff Institute in October and November 2016. Our aim was to understand the implications of recent developments in Goodwillie calculus for unstable homotopy theory. More specifically, we studied unstable v_n-periodic homotopy theory, which is a localization of homotopy theory focusing on certain periodic phenomena in homotopy groups. The case $n = 0$ corresponds to rational homotopy theory. For $n > 0$, we pursued models of these v_n-periodic homotopy theories related to Lie algebras and commutative coalgebras in the category of spectra. These models are analogous to those of Quillen and Sullivan for rational homotopy theory.

Seminars. During our stay we organized the 'Seminar on functor calculus and chromatic methods', with research talks by Lukas Brantner, Dustin Clausen, Wolfgang Lück, Tomer Schlank, and Sarah Yeakel. Moreover, Greg Arone gave a very successful series of three lectures on various applications of Goodwillie calculus to homotopy theory, including chromatic homotopy theory. Other than giving great talks, each of our visitors provided valuable input to our research projects.

Research. The work we did during our time at the HIM has sparked a collaboration that is still ongoing. So far this has resulted in the following:

(1) In the recent preprint [1], we show that the Bousfield-Kuhn functor is part of a monadic adjunction, which shows that v_n-periodic unstable homotopy theory can be described as a homotopy theory of algebras for a monad on the category of $T(n)$-local spectra.
(2) The soon to appear [2] will show that the monad of (1) is in fact the free Lie algebra monad, giving a model for v_n-periodic unstable homotopy theory generalizing Quillen's Lie algebra model for rational homotopy theory.
(3) Mathew proved a useful nilpotence lemma which allows for the interchange of certain limits with Goodwillie's n-excisive approximation functor P_n in the setting of telescopic homotopy theory. Dually, his results allow the interchange of certain colimits with the dual approximations P^n studied by McCarthy. These results are also used in [2].
(4) In rational homotopy, any loop space is automatically an infinite loop space. Bousfield proved an analogue of this statement in v_1-periodic homotopy theory for spaces with finite p-exponent, which Meier generalized to a statement applying to v_n-periodic homotopy theory for any n.
(5) We have an ongoing research project on costabilization of homotopy theories, which intends to give a general framework in which the Bousfield-Kuhn functor can be characterized as the costabilization of v_n-periodic unstable homotopy theory. These methods should also be useful in the context of homotopy theories of algebras (and coalgebras) over operads (resp. cooperads).

References

[1] Eldred, R, and Heuts, G, and Mathew, A, and Meier, L. "Monadicity of the Bousfield-Kuhn functor", arXiv:1707.05986.

[2] Heuts, G. "Lie algebras and v_n-periodic spaces", to appear.